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10MAT31

Third Semester B.E. Degree Examination, June/July 2014
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Find Fourier series of $f(x) = 2\pi x - x^2$ in $[0, 2\pi]$. Hence deduce $\sum_1^\infty \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. Sketch the graph of f(x). **(07 Marks)**

b. Find Fourier cosine series of $f(x) = \sin\left(\frac{m\pi}{\ell}\right)x$, where m is positive integer. **(06 Marks)**

c. Following table gives current (A) over period (T):

A (amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98
t (sec)	0	T/6	T/3	T/2	2T/3	5T/6	T

Find amplitude of first harmonic. **(07 Marks)**

2 a. Find Fourier transformation of $e^{-a^2x^2}$ $(-\infty < x < \infty)$ hence show that $e^{-x^2/2}$ is self reciprocal. **(07 Marks)**

b. Find Fourier cosine and sine transformation of

$$f(x) = \begin{cases} x & 0 < x < a \\ 0 & x \geq a \end{cases}$$ **(06 Marks)**

c. Solve integral equation $\int_0^\infty f(x)\cos sx\,dx = \begin{cases} 1-s & 0 < s < 1 \\ 0 & s \geq 1 \end{cases}$. Hence deduce $\int_0^\infty \frac{1-\cos x}{x^2}dx = \frac{\pi}{2}$. **(07 Marks)**

3 a. Find various possible solution of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ by separable variable method. **(07 Marks)**

b. Obtain solution of heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial t^2}$ subject to condition u(0, t) = 0, u(ℓ, t) = 0, u(x, 0) = f(x). **(06 Marks)**

c. Solve Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to condition u(0, y) = u(ℓ, y) = u(x, 0) = 0; $u(x, a) = \sin\left(\frac{\pi x}{\ell}\right)$. **(07 Marks)**

4 a. The revolution (r) and time (t) are related by quadratic polynomial $r = at^2 + bt + c$. Estimate number revolution for time 3.5 units, given

Revolution	5	10	15	20	25	30	35
Time	1.2	1.6	1.9	2.1	2.4	2.6	3

(07 Marks)

b. Solve by graphical method,
Minimize $Z = 20x_1 + 10x_2$ under the constraints $2x_1 + x_2 \geq 0$; $x_1 + 2x_2 \leq 40$; $3x_1 + x_2 \geq 0$; $4x_1 + 3x_2 \geq 60$; $x_1, x_2 \geq 0$. **(06 Marks)**

c. A company produces 3 items A, B, C. Each unit of A requires 8 minutes, 4 minutes and 2 minutes of producing time on machine M_1, M_2 and M_3 respectively. Similarly B requires 2, 3, 0 and C requires 3, 0, 1 minutes of machine M_1, M_2 and M_3. Profit per unit of A, B and C are Rs.20, Rs.6 and Rs.8 respectively. For maximum profit, how many number of products A, B and C are to be produced? Find maximum profit. Given machine M_1, M_2, M_3 are available for 250, 100 and 60 minutes per day. **(07 Marks)**

PART – B

5 a. By relaxation method, solve $-x+6y+27z=85$, $54x+y+z=110$, $2x+15y+6z=72$. **(07 Marks)**

b. Using Newton Raphson method derive the iteration formula to find the value of reciprocal of positive number. Hence use to find $\frac{1}{e}$ upto 4 decimals. **(06 Marks)**

c. Using power rayley method find numerical largest eigen value and corresponding eigen vector for $\begin{bmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 2 & 1 & 10 \end{bmatrix}$ using $(1, 1, 0)^T$ as initial vector. Carry out 10 iterations. **(07 Marks)**

6 a. Fit interpolating polynomial for f(x) using divided difference formula and hence evaluate f(z), given f(0) = - 5, f(1) = -14, f(4) = -125, f(8) = -21, f(10) = 355. **(07 Marks)**

b. Estimate t when f(t) = 85, using inverse interpolation formula given : **(06 Marks)**

t	2	5	8	14
f(t)	94.8	87.9	81.3	68.7

c. A solid of revolution is formed by rotating about x-axis, the area between x-axis, lines x = 0, x = 1 and curve through the points with the following co-ordinates.

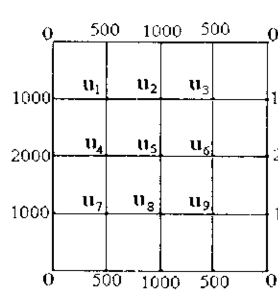
x	0	1/6	2/6	3/6	4/6	5/6	1
y	0.1	0.8982	0.9018	0.9589	0.9432	0.9001	0.8415

by Simpson's $3/8^{th}$ rule, find volume of solid formed. **(07 Marks)**

7 a. Using the Schmidt two-level point formula solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ under the conditions $u(0, t) = u(1, t) = 0$; $t \geq 0$; $u(1, 0) = \sin \pi x \quad 0 < x < 1$, take $h = \frac{1}{4}$ $\alpha = \frac{1}{6}$. Carry out 3 steps in time level. **(07 Marks)**

b. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to $u(0, t) = u(4, t) = u_t(x, 0) = 0$, $u(x, 0) = x(4-x)$ take h = 1 k = 0.5. **(06 Marks)**

c. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the square mesh. Carry out 2 iterations. **(07 Marks)**



8 a. State and prove recurrence relation of f-transformation hence find $Z_T(n)$, $Z_T(n^2)$. **(07 Marks)**

b. Find $Z_T[e^{n\theta} \cosh n\theta - \sin(nA+\theta) + n]$. **(06 Marks)**

c. Solve difference equation $u_{n+2} + 6u_{n+1} + 9u_n = n2^n$ given $u_0 = u_1 = 0$. **(07 Marks)**

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